Overview Of Lecture

- Impedance Conversion
  - Up Converter
  - Down Converter
  - Pi-Section

- Constant Resistance Networks
  - Attributes
  - Minimal Phase Filters
  - Non-Minimal Phase Filters
  - All Pass Networks
  - Design Example
Generalization Of Interstage Filter Problem

- **System Diagram**

- **Design Issues**
  - Desire $Z_{in}(j\omega_o) = R_s$  
    - Maximum Power Transfer From Signal Source To Filter Input Port  
    - Generally Accomplished At Only A Tuned Center Frequency, $\omega_o$  
    - Generally Accomplished For Only A Reasonably Narrow Passband  
    - Filter Is Usually Lossless Topology In RF Communication Systems  
  - Convert $R_i$ At Output Port -To- $R_s$ At Input Port  
    - Up Converter Implies $R_s > R_i$  
    - Down Converter Signifies $R_s < R_i$

- **Design Specifications**
  - Tuned Center Frequency, $\omega_o$  
  - 3-dB Bandwidth Or Equivalently, Effective System $Q$  
  - Up/Down Impedance Conversion Factor, $K_z$
Up Conversion Interstage Filter (UCF)

- **Circuit Topology**
  - **Admittance,** $Y_l(s)$
    - Radial Center Frequency Is $\omega_o$
    - Load $Q$
      \[ Q_l = \frac{\omega_o L}{R_l + R_c} \]
  - Implication Is Shunt $RL$ Circuit At Tuned Center Frequency

- **Admittance Function**
  \[ Y_l(j\omega) = \frac{1}{\frac{R_l + R_c}{R_l + R_c + j\omega L}} = \frac{R_l + R_c - j\omega L}{(R_l + R_c)^2 + (\omega L)^2} = \frac{1 - jQ_l \left(\frac{\omega}{\omega_o}\right)}{\left(R_l + R_c\right) \left[1 + Q_l^2 \left(\frac{\omega}{\omega_o}\right)^2\right]} \]
UCF Circuit Model

- At Center Frequency

\[ Y_L(j\omega_o) = \frac{1}{(1 + Q_l^2)(R_l + R_c)} + \frac{Q_l^2}{j\omega_o(1 + Q_l^2)L} \]

- Effective Shunt Inductance, \( L_{eff} \)

\[ L_{eff} = \left( \frac{1 + Q_l^2}{Q_l^2} \right) L \]

- Effective Shunt Resistance, \( R_{eff} \)

\[ R_{eff} = \left( 1 + Q_l^2 \right)(R_l + R_c) \]

- Resonance, \( \omega_o \)

\[ \omega_o = \frac{1}{\sqrt{L_{eff}C}} = \frac{Q_l}{\sqrt{(1 + Q_l^2)LC}} \]

- Impedance Conversion, \( K_z \)

\[ K_z = \frac{R_s}{R_l} = \frac{R_{eff}}{R_l} = \left( 1 + \frac{R_c}{R_l} \right) \left( 1 + Q_l^2 \right) \]

- Model At Frequency \( \omega_o \)
UCF Circuit Design Example

- **Specifications**
- **Calculations**
  - Quality Factor:
    \[
    Q_l = \sqrt{\frac{K_z}{I + \frac{R_c}{R_l}}} - 1 = \sqrt{\frac{R_s}{R_l} \left(1 + \frac{R_c}{R_l}\right)} - 1 = 1.504
    \]
  - Inductance:
    \[
    L = \left(R_l + R_c\right)\left(\frac{Q_l}{\omega_o}\right) = 2.04 \text{ nH}
    \]
  - Capacitance:
    \[
    C = \frac{Q_l^2}{\left(1 + Q_l^2\right)\omega_o^2 L} = 1.18 \text{ pF}
    \]
- Result (ohms, pF, nH)

Load Resistance, \(R_l\): 20 ohms
Source Resistance, \(R_s\): 75 ohms
Tuned Matching Frequency, \(f_o\): 2.7 GHz
Estimated Inductor Resistance, \(R_c\): 3 ohms
Up Converter Filter Simulation

Filter Input Impedance

Impedance (Ohms)

75 Ohms

Real Part

Imaginary Part

FREQUENCY (Hz)

2.72 GHz

HSPICE
Comments On UCF Design Example

• Filter Satisfies Design Requirements
  ▪ Imaginary Part Of Input Impedance Is Zero At 2.75 GHz
  ▪ Real Part Of Input Impedance Is 75 Ohms At 2.75 GHz

• Observations
  ▪ Imaginary Part Of Input Impedance
    ◆ Positive Below 2.75 GHz Resonance, Indicating Inductive Impedance
    ◆ Negative Above 2.75 GHz, Indicating Capacitive Input Impedance
  ▪ Real Part Of Input Impedance
    ◆ Value Is As Expected At Resonance
    ◆ Peak Occurs Beyond Resonance

• It Can Be Shown (favorite professorial line)
  ▪ Real Part Peak Occurs At Frequency $\omega_x$
  ▪ Peak Real Part Is A Value, $R_x$
  ▪ Large Q Means Peaking Coincides With Resonance \(\omega_x = \omega_o\) And
    \(R_x = \text{Re}[Z_{in}(j\omega_o)]\)

\[
\omega_x = \omega_o \sqrt{1 + \frac{1}{2Q_l^2}}
\]

\[
R_x = \text{Re}\left[Z_{in}(j\omega_o)\right]\left(\frac{Q_l^2 + 1}{Q_l^2 + 3/4}\right)
\]
Up Converter Filter Analysis

- **Input Impedance**
  \[ R \triangleq R_l + R_c \]
  \[ Z_{in}(j\omega) = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC} \triangleq R_{in} + jX_{in} \]

- **Real Part Input Impedance**
  \[ R \_{in} = \frac{R}{R} \left( 1 + Q_l^2 \right)^2 \]
  \[ \left( 1 + Q_l^2 \right)^2 - \left( 1 + 2Q_l^2 \right) \left( \frac{Q_l}{\omega_o} \right)^2 + \left( \frac{Q_l}{\omega_o} \right)^4 \]

- **Imaginary Part Input Impedance**
  \[ X \_{in} = \frac{Q_l^2 \left( 1 + Q_l^2 \right) \left[ 1 - \left( \omega/\omega_o \right)^2 \right] \left( \omega/\omega_o \right)}{\omega_o L \left( 1 + Q_l^2 \right)^2 - \left( 1 + 2Q_l^2 \right) \left( \frac{Q_l}{\omega_o} \right)^2 + \left( \frac{Q_l}{\omega_o} \right)^4} \]
UCF Input Resistance Characteristics

![Graph showing normalized real part versus normalized frequency for different quality factors.]

Normalized Real Part

Normalized Frequency, \( \frac{\omega}{\omega_0} \)

- \textit{Quality Factor} = 6
- \textit{Quality Factor} = 4
- \textit{Quality Factor} = 2
UCF Input Reactance Characteristics

Normalized Reactance vs. Normalized Frequency, \((\omega/\omega_o)\)

- **Quality Factor = 6**
- **Quality Factor = 3**
- **Quality Factor = 2**
Down Conversion Interstage Filter (DCF)

- **Circuit Topology**
- **Admittance, \( Z_l(s) \)**
  - Radial Center Frequency Is \( \omega_o \)
  - Load \( Q_l = \frac{\omega_o R_l C}{C} \)
  - Load Impedance Function
  
  \[
  Z_l(j\omega) = \frac{R_l}{1 + j\omega R_l C} = R_o(\omega) + \frac{1}{j\omega C_o(\omega)}
  \]

- **Parameters**
  
  \[
  R_o(\omega) = \frac{R_l}{1 + Q_l \left( \frac{\omega}{\omega_o} \right)^2}
  \]

  \[
  C_o(\omega) = \frac{1 + Q_l \left( \frac{\omega}{\omega_o} \right)^2}{C}
  \]

- **Implication Is Series**
  RC Circuit To Be Tuned To Center Frequency By Inductance \( L \)
Down Converter Interstage Filter Analysis

- **Tuning Frequency**
  \[ \omega_o = \frac{1}{\sqrt{LC_o(\omega_o)}} = \frac{Q_l}{\sqrt{(1 + Q_l^2)}LC} \]

- **Tuned Input Resistance**
  \[ R_{\text{eff}} = R_c + R_o(\omega_o) = R_c + \frac{R_l}{1 + Q_l^2} \]

- **Resistance Conversion**
  \[ Q_l = \sqrt{\frac{R_l}{R_s - R_c}} - 1 = \sqrt{\frac{K_z}{1 - R_c/R_s}} - 1 \]

- For Input Matching, \( R_{\text{eff}} = R_s \)
- Down Conversion Factor Is \( K_z = R_l/R_s \)
- Note That Small Inductor \( Q \) (Large \( R_c \)) Promotes The Need For Large Load Quality Factor, \( Q_l \)
DCF Circuit Design Example

- **Specifications**
  - Load Resistance, $R_l$: 250 ohms
  - Source Resistance, $R_s$: 50 ohms
  - Tuned Matching Frequency, $f_0$: 1.0 GHz
  - Estimated Inductor Resistance, $R_c$: 4 ohms

- **Calculations**

  - **Quality Factor**
    
    \[
    Q_l = \sqrt{\frac{R_l}{R_s - R_c}} - 1 = \sqrt{\frac{K_z}{1 - R_c/R_s}} - 1 = 2.106
    \]

  - **Capacitance**
    
    \[
    C = \frac{Q_l}{\omega_o R_l} = 1.34 \text{ pF}
    \]

  - **Inductance**
    
    \[
    L = \frac{Q_l^2}{\left(1 + Q_l^2\right)\omega_o^2 C} = 15.42 \text{ nH}
    \]

- **Result (ohms, pF, nH)**

  ![Diagram of the circuit design with the calculated values integrated into the diagram.](image-url)
Down Converter Filter Simulation

![Graph showing impedance and frequency relationship]

**IMPEDANCE (Ohms)**

-400 to 400

**REAL PART**

**IMAGINARY PART**

**FREQUENCY (Hz)**

$10^8$ to $10^{10}$

*HSPICE*
Comments On DCF Design Example

- **Filter Satisfies Design Requirements**
  - Imaginary Part Of Input Impedance Is Zero At 1.0 GHz
  - Real Part Of Input Impedance Is 50 Ohms At 1.0 GHz

- **Observations**
  - Imaginary Part Of Input Impedance
    - Negative Below 1.0 GHz Resonance, Indicating Capacitive Impedance
    - Positive Above 1.0 GHz, Indicating Inductive Input Impedance
  - Real Part Of Input Impedance
    - Value Is As Expected At Resonance
    - Monotone Decreasing Function Of Frequency

- **Detailed Analysis**
  - Deduce Expression for $Z_{\text{in}}(j\omega)$
  - Decompose Impedance Into Real And Imaginary Parts

\[
Z_{\text{in}}(j\omega) = j\omega L + \frac{R_l}{R_c} + \frac{R_l}{1 + j\omega R_l C} = R_{\text{in}}(\omega) + jX_{\text{in}}(\omega)
\]
Detailed Down Converter Filter Analysis

- **Real Part Input Impedance**

\[
\frac{R_{in}(\omega)}{R_c + R_l} = 1 + \left(\frac{Q_l \omega}{\omega_o}\right)^2 \left(\frac{R_c}{R_c + R_l}\right)
\]

- **Imaginary Part Input Impedance**

\[
\frac{X_{in}(\omega)}{R_l} = \left(\frac{Q_l^2}{Q_l^2 + 1}\right) \frac{Q_l \omega}{\omega_o} \left[1 + \left(\frac{Q_l \omega}{\omega_o}\right)^2\right] \left[\left(\frac{\omega}{\omega_o}\right)^2 - 1\right]
\]
Down Converter Filter Real Part Response

Resistance Normalization Is With Respect To \((R_c + R_L)\)

Normalized Input Resistance

Normalized Frequency, \((\omega/\omega_o)\)

Load Quality Factor = 2
Load Quality Factor = 4
Load Quality Factor = 6
Down Converter Filter Reactance Response

Load Quality Factor = 2
Load Quality Factor = 4
Load Quality Factor = 6

Resistance Normalization Is With Respect To $R_l$
Pi-Section Interstage Matching Filter

- **Attributes**
  - Three Design Degrees Of Freedom \((L, C_a, C_b)\)
  - Allows For Up Or Down Conversion

- **Decompose Inductance**
  - \(L = L_a + L_b\)
  - Seen As Cascade Of Up And Down Conversion Filters
  - Lump Inductor Resistance With Inductance \(L_a\)
  - Start With Load Impedance
    - Write As Series RC
      \[Z_l(j\omega) = R_o(\omega) + 1/j\omega C_o(\omega)\]
    - Load Quality Factor
      \[Q_o = \omega_o R_l C_b\]
    - Tuned Center Frequency Is \(\omega_o\)
Pi-Section Filter Analysis

- **Load Impedance**

\[ Z_l(j\omega) = \frac{R_l}{1 + j\omega R_l C_b} = R_{o}(\omega) + 1/j\omega R_l C_{o}(\omega) \]

- **Resistance And Capacitance Expressions**

\[ R_{o}(\omega) = \frac{R_l}{1 + \left(\frac{Q_o \omega}{\omega_o}\right)^2} \quad C_{o}(\omega) = \frac{\omega_o}{Q_o} \left[ 1 + \left(\frac{\omega}{\omega_o}\right)^2 \right] C_b \]

- **DCF Resonance**

\[ \omega_o = \frac{1}{\sqrt{L_b C_{o}(\omega_o)}} = \frac{Q_o}{\sqrt{\left(1 + Q_o^2\right)L_b C_b}} \]
Pi-Section Filter Analysis . . . Cont’d

- **Admittance,** $Y_i(j\omega)$
  - Decompose Admittance Into Real And Imaginary Functional Components
  - Too Cumbersome To Achieve As General Frequency Function
  - Evaluate Real And Imaginary Parts At Frequency $\omega_o$
  - Make Use Of Resonance Condition With Respect To $L_b$
  - Introduce Input (DCF) Quality Factor, $Q_i$

- **Real & Imaginary Parts Of** $Y_i(j\omega)$

$$
\begin{align*}
\text{Re}[Y_i(j\omega_o)] & \triangleq \frac{I}{R_i(\omega_o)} = \frac{I}{\left(1 + Q_i^2\right) \left[\frac{1}{R_c + R_o(\omega_o)}\right]} \\
\text{Im}[Y_i(j\omega_o)] & = -\frac{Q_i^2}{\left(1 + Q_i^2\right) \left[\frac{1}{\omega_o L_a} + \frac{1}{R_c + R_o(\omega_o)}\right]}
\end{align*}
$$

$$
\begin{align*}
Q_i & \triangleq \frac{\omega_o L_a}{R_c + R_o(\omega_o)} \\
L_i(\omega_o) & = \left(\frac{1 + Q_i^2}{Q_i^2}\right) L_a \\
R_o(\omega_o) & = \frac{R_l}{1 + Q_o^2}
\end{align*}
$$
Pi Filter Center Frequency Model

- **Original Form**
- **Tuned Center Frequency Model**

**Resonance**
- Must Be Same As Resonance With Respect To Inductance \( L_b \)
- Center Frequency
- Converted Input Resistance

\[
R_{in}(\omega_o) \equiv R_i(\omega_o) = \left(1 + Q_i^2\right)R_c + \left(\frac{l + Q_i^2}{1 + Q_o^2}\right)R_l
\]

\[
\omega_o = \frac{1}{\sqrt{L_i(\omega_o)C_a}} = \frac{Q_i}{\sqrt{(1 + Q_i^2)L_aC_a}}
\]
Approximate Pi-Section Filter Bandwidth

- **Shunt Q**
  \[
  Q_s = \omega_o \left[ \frac{R_s}{R_i(\omega_o)} \right] C_a
  \]

- **Shunt Q Related To \( Q_i \)**
  \[
  Q_s = \omega_o \frac{R_i(\omega_o)}{2} C_a = \frac{\left( 1 + Q_i^2 \right) \left[ R_c + R_o(\omega_o) \right]}{2 \omega_o L_i(\omega_o)}
  \]
  \[
  \approx \frac{\left( 1 + Q_i^2 \right) \left[ R_c + R_o(\omega_o) \right] Q_i^2}{2 \omega_o \left( 1 + Q_i^2 \right) L_a}
  \equiv \frac{Q_i}{2}
  \]

- **3-dB Bandwidth**
  - Approximate Bandwidth
    - Effective Shunt Inductance Is Not Constant, But Is Frequency Variant
    - Effective Shunt Capacitance Is Not Constant, But Is Frequency Variant
  - Approximate Bandwidth Is Additional Designable Metric Not Afforded By Either Up Or Down Conversion Filters

\[ B \approx \frac{\omega_o}{Q_s} = \frac{2 \omega_o}{Q_i} = \frac{2}{R_i(\omega_o) C_a} \]
Pi-Filter Section Design Example

- **Specifications**
  - Load Resistance, $R_l$: 20 ohms
  - Shunt Load Capacitance, $C_l$: 200 fF
  - Source Resistance, $R_s$: 50 ohms
  - Tuned Matching Frequency, $f_0$: 2.2 GHz
  - 3-dB Filter Bandwidth: 800 MHz
  - Estimated Inductor Resistance, $R_c$: 1 ohm

- **Calculations**
  - **Input Quality Factor**
    $$Q_i = \frac{2\omega_0}{B} = 5.50$$
  - **Capacitance, $C_a$**
    $$C_a = \frac{Q_i}{\omega_0 R_i(\omega_0)} = 7.958\ \text{pF}$$
  - **Inductance, $L_a$**
    $$L_a = \frac{Q_i^2}{\left(1 + Q_i^2\right)\omega_0^2 C_a} = 636.6\ \text{pH}$$

- **Computations Continue On Following Slide**
Pi-Section Design Example . . . Cont’d

From Previous Slide
- $R_{in}(\omega_o) = R_s = 50$ Ohms
- Calculate Load $Q$, $Q_o$
- Calculate $C_b$
\[ C_b = \frac{Q_o}{\omega_o R_i} = 20.57 \text{ pF} \]
- Calculate $L_b$
- Calculate Total Inductance, $L$
\[ L = L_a + L_b = 883.4 \text{ pH} \]

Final Design
- Reduce $C_b$ By 200 fF To Account For Load Capacitance
- Elements In Ohms, pF, And pH

\[
Q_o = \sqrt{\frac{(1 + Q_i^2) R_i}{R_s - (1 + Q_i^2) R_c}} - 1 = 5.686
\]
\[
L_b = \frac{Q_o^2}{\left(1 + Q_o^2\right) \omega_o^2 C_b} = 246.8 \text{ pH}
\]
Pi-Section Simulated Input Impedance

Frequency (Hz)

Impedance (ohms)

Real Part

Imaginary Part

50 Ohms

2.2 GHz
Comments On Pi-Section Design

- **Filter Satisfies Design Requirements**
  - Imaginary Part Of Input Impedance Is Zero At 2.2 GHz
  - Real Part Of Input Impedance Is 50 Ohms At 2.2 GHz

- **Observations**
  - Imaginary Part Of Input Impedance
    - Negative Above 2.2 GHz, Indicating Capacitive Impedance
    - Negative At Low Frequencies, Indicating Capacitive Impedance
    - Inductive In Neighborhood Of Resonance
  - Imaginary Part Is Zero At Two Signal Frequencies

- **Fundamental Limitation**

  \[
  R_i(\omega_o) = R_s = \left(1 + Q_i^2\right) R_c + \left(\frac{1 + Q_i^2}{1 + Q_o^2}\right) R_l \Rightarrow R_c \leq \frac{R_s}{1 + Q_i^2}
  \]

  - Implies Bandwidth And Inductor Q Constraints
  - Most Severe For Down Conversion Application
**Constant Resistance Interstage Filters**

- **Simplified Design Strategy**
  - Individual Subcircuits
    - Each Terminated In Resistance $R$
    - Each Delivers Frequency Invariant Input Impedance, $Z_{in}(s)$, Identical To $R$
    - Transfer Functions Evaluated With Load Termination Of $R$

- **Resultant System Gain**
  - Proportional To Product Of Individual Transfer Functions
  - Complex Function Can Be Decomposed Into Simpler Functions
  - Overall Transfer Function

$$H(s) = \frac{V_o}{V_s} = \left( \frac{R}{R + R_s} \right) H_1(s) H_2(s)$$
**Constant Resistance Tee-Section Filter**

- **Circuit**

- **Criterion For** $Z_{in}(s) \equiv R$

- **Gain Under Constant Resistance Constraint**

- **Comments**
  - Port Gain Magnitude Is Always Less Than Unity
  - Tee Structure Is Not Lossless; $Z_a$ And $Z_b$ Are Desirably Lossless
  - Satisfaction Of Desired I/O Transfer Function
    - Reduces To Determination Of Impedance $Z_a$
    - Impedance $Z_b$ Is Found As Inverse Of $Z_a$ With Respect To $R^2$
      - Capacitive $Z_a$ Becomes Inductive $Z_b$ (Highpass Network)
      - Inductive $Z_a$ Becomes Capacitive $Z_b$ (Lowpass Network)

\[ H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{Z_a}{R}} \]

\[ Z_b = \frac{R^2}{Z_a} \]
Constant Resistance El-Section Filter

- **Circuit**
- **Criterion For** \( Z_{in}(s) \equiv R \)
  \[
  Z_a Z_b = R^2;
  \]
- **Gain For Constant Resistance Constraint**
  \[
  H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{Z_a}{R}}
  \]
- **Comments**
  - Port Gain Magnitude Is Less Than Unity
  - Ell Structures Are Not Lossless; \( Z_a \) And \( Z_b \) Are Desirably Lossless
  - Satisfaction Of Desired I/O Transfer Function
    - Reduces To Determination Of Impedance \( Z_a \)
    - Impedance \( Z_b \) Is Found As Inverse Of \( Z_a \) With Respect To \( R^2 \)
Constant Resistance Bridge Architecture

- **Circuit**
- **Criterion For** $Z_{in}(s) \equiv R$:
  \[ Z_a Z_b = R^2 \]
- **Gain Under Constant Resistance**
  \[ H(s) = \frac{V_o}{V_i} = \frac{1 - Z_a/R}{1 + Z_a/R} \]

- **Comments**
  - **Allpass Network When** $Z_a$ **Is Lossless**
    - Gain Magnitude Equal One For All Signal Frequencies
    - Phase Angle Is Twice That Contributed By Pole
    - Large Envelope Delays Are Possible
  - **Disadvantage Is Lack Of Common I/O Ground**
    - Can Be Mitigated For Second Order $Z_a$
    - Cannot Be Transformed Into Common Ground Structure For Single Order $Z_a$
    - Output Can Be Applied To A Differential -To- Single Ended Converter
First Order Bridge Filter

- **Transfer Relationship**
  \[ H(s) = \frac{V_o}{V_i} = \frac{\omega_h - s}{\omega_h + s} = \frac{1 - s/\omega_h}{1 + s/\omega_h} = \frac{1 - Z_a/R}{1 + Z_a/R} \]

- **Impedances**
  \[ Z_a = \frac{R_s}{\omega_h} \quad Z_b = \frac{R^2}{Z_a} = \frac{l}{s/R\omega_h} \]

- **Realization**
  - **First Order Network**
    - Four Energy Storage Elements
    - One Independent Initial Condition
  - Right Half Plane Zero
  - Constant Input Impedance
  - Phase Response
    \[ \varphi(\omega) = -2 \tan^{-1} \left( \frac{\omega}{\omega_h} \right) \]
  - Envelope Delay
    - Enhanced By Right Half Plane Zero
    - Zero Frequency Value Is \( \frac{2}{\omega_h} \)
**Second Order Bridge Filter**

- **Transfer Relationship**

\[
H(s) = \frac{V_o}{V_i} = \frac{1 - \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2} = \frac{1 - \frac{s/Q\omega_o}{1 + \left(\frac{s}{\omega_o}\right)^2}}{1 + \frac{s/Q\omega_o}{1 + \left(\frac{s}{\omega_o}\right)^2}}
\]

- **Impedances**

\[
Z_a = \frac{Rs/Q\omega_o}{1 + \left(\frac{s}{\omega_o}\right)^2} = \frac{1}{\frac{Q\omega_o}{Rs} + \frac{Qs}{R\omega_o}}
\]

\[
Z_b = \frac{R^2}{Z_a} = \frac{QR\omega_o}{s} \left[1 + \left(\frac{s}{\omega_o}\right)^2\right] = \frac{QR\omega_o}{s} + \frac{QRs}{\omega_o}
\]

- **Realization On Following Slide**
Circuit

Comments

- Allpass Configuration
- Lacks A Common I/O Ground
- Generalized Realization For Transfer Function \( H(s) \)

\[
H(s) = \frac{V_o}{V_i} = \frac{1 - \frac{s}{Q\omega_o} + \left( \frac{s}{\omega_o} \right)^2}{1 + \frac{s}{Q\omega_o} + \left( \frac{s}{\omega_o} \right)^2}
\]
**Bridge-To-Bridged-Tee Filter Conversion**

**Topological Conversion**

1. Equate Two Port Parameter Matrices Of Networks
2. Invoke Condition Of Constant Input Resistance
3. Converts Non-Common Ground Architecture To Common Ground Topology
4. Two Cases Materialize, As Documented On Following Slides
**Bridged-Tee Conversion . . . Case #1**

- **The Case Of** $L_b / L_a \geq 1$

- **I/O Terminal Characteristics Of Tee Network**
  - Identical To Those Of Bridge Configuration
  - Coupled Inductor Can Be Used To Replace Inductor Tee

\[ Z_{in}(s) = R \]
Bridged-Tee Conversion . . . Case #2

The Case Of $C_a/C_b \geq 1$

I/O Terminal Characteristics Of Tee Network
- Identical To Those Of Bridge Configuration
- Uses Two Uncoupled Inductors
Third Order Lowpass Bessel Filter

- **Port Transfer Function**
  - DC Port Gain Is One
  - Specifications
    - Load Termination Is 300 $\Omega$, Source Termination Is 50 $\Omega$
    - Zero Frequency Delay Is 25 pSEC
    - Unmatched Input Port Implies Port Gain Multiplied By $(300/350) = 6/7$
  - Normalization
    - 300 Ohm Impedances Normalize To 1 Ohm
    - Radial Frequency Of $1/T_{\text{do}} = 40$ GRPS Normalizes To 1 RPS (RPS Is Radians -Per- Second & GRPS Is Giga-Radians -Per- Second)
    - Normalizing Parameters
      - Inductance: $L_\alpha = Z_\alpha / \omega_\alpha = 7.5 \text{ nH}$
      - Capacitance: $C_\alpha = 1/ \omega_\alpha Z_\alpha = (250/3) \text{ fF}$

- **Design Strategy**
  - Factor Characteristic Polynomial To Expose Second Order Function
  - Augment Transfer Function By Pole Zero Introduction
  - Realize As Cascade Of Constant Resistance Structures

\[
H(s) = \frac{V_o}{V_i} = \frac{15}{s^3 + 6s^2 + 15s + 15}
\]
Lowpass Bessel Filter Design Step 1

- **Factor Transfer Function**

\[
\frac{V_o}{V_i} = \frac{15}{s^3 + 6s^2 + 15s + 15} = \frac{15}{(s + 2.322)\left(s^2 + 3.678s + 6.459\right)}
\]

- **Augment Transfer Function**

\[
\frac{V_o}{V_i} = \frac{15K(s + a)}{(s + 2.322)\left(s^2 + 3.678s + 6.459\right)(s + a)}
= \frac{6.459K/a}{s^2 + 3.678s + 6.459} \times \frac{a}{s + a} \times \frac{2.322}{s + 2.322}
\]

- **Engineering Strategy**
  - Overall Filter Is Cascade Of Three Filter Sections, As Indicated
  - Parameter “a” Is An Arbitrary, But Positive, Number, Chosen To Assure Physical Realizability Of First Filter Section
  - DC Gain Realized To Within Factor Of \( K = a/6.459 \) To Ensure Realizability
Comments On Bessel Design Step 1

- Transfer Relationship
  \[
  \frac{V_o}{V_i} = \frac{(6.459K/a)(s + a)}{\left(s^2 + 3.678s + 6.459\right)} \times \frac{a}{(s + a)} \times \frac{2.322}{(s + 2.322)}
  \]

- Realizability Constraints
  - Constant Resistance Sections
    - Constant Multiplier Is \((6.459K/a)(2.322) = 15K\), As Required
    - Constant Factor \(K\) Introduced Because Realization With Tee Or El Sections Requires Port Transfer Function Of Form:
      \[
      H(s) = \frac{1}{1 + Z \frac{a}{R}}
      \]
      Thus: \(6.459K/a = 1 \Rightarrow K = a/6.459\)
      - Small \(a\) To Be Avoided To Preclude Strong Attenuation
    - Last Two Terms Readily Produce Desired Transfer Function Form
  - Parameter \(a\)
    - Chosen To Ensure Positive Circuit Elements In Realization
    - Critical Only In The Sense Of Impact On Port Gain Attenuation
    - Plausible Values Of \(a\) Determined Through Continued Fraction Expansion Of First Transfer Relationship
Lowpass Bessel Filter Design Step 2

- Continued Fraction Expansion Of Filter Section 1 Transfer Function

\[
\frac{s^2 + 3.678s + 6.459}{s + a} = s + (3.678 - a) + \frac{6.459 - a(3.678 - a)}{s + a}
\]

- “a” Must Be Chosen Such That \( (3.678 - a) \geq 1 \); Ensures Attainment Of Requisite Transfer Function Form
- Also, \( 6.459 - a(3.678 - a) \) Must Be Positive
- Latter Constraint Implies \( (a - 1.839)^2 + 3.077 > 0 \), Which Is Satisfied For All “a”
- Choose \( a = 2.5 \) (Avoid Setting \( a = 2.678 \), Which Satisfies The Foregoing Precise Unity Constraint, Because Of Possible Round Off Computation Errors)
- Note: \( K = a/6.459 = 2.5/6.459 = 0.3871 \)

- Filter Section 1 Result

\[
\frac{(s + 2.5)}{(s^2 + 3.678s + 6.459)} = \frac{1}{1 + s + 0.178 + \frac{1}{0.2846s + 0.7114}}
\]
Lowpass Bessel Filter Design Step 3

- **Impedance** $Z_a$ of Section 1 ($R = 1$ Ohm)

$$Z_a = Z_a = s + 0.178 + \frac{1}{0.2846s + 0.7114}$$

- **Impedance** $Z_b$ of Section 1

$$Z_b = \frac{R^2}{Z_a} = \frac{1}{Z_a} = s + 0.178 + \frac{1}{0.2846s + 0.7114}$$

- Can Be Realized By Either EI Circuit Documented Previously
Lowpass Bessel Filter Design Step 4

**Filter Section 2**

\[
\frac{a}{s + a} = \frac{l}{1 + s/a} = \frac{l}{1 + s/2.5}
\]

\[
\frac{Z_a}{R} = \frac{Z_a}{2.5} = \frac{s}{2.5}
\]

\[
Z_b = \frac{R^2}{Z_a} = \frac{l}{Z_a} = \frac{l}{s/2.5}
\]

**Filter Section 3**

\[
\frac{2.322}{s + 2.322} = \frac{l}{1 + s/2.322} = \frac{l}{1 + 0.4307s}
\]

\[
\frac{Z_a}{R} = Z_a = 0.4307s
\]

\[
Z_b = \frac{R^2}{Z_a} = \frac{l}{Z_a} = \frac{l}{0.4307s}
\]
Bessel Filter Example Realization

- **Normalized Form**
  - Resistors In Ohms
  - Inductors In Henries
  - Capacitors In Farads

- **Structures**
  - EI Topology Used For Filter Sections 1 And 2
  - EI Topology Used For Filter Section 3
  - Choice Is Somewhat Arbitrary; Often Dictated By Ease Of Physical Realizability Or The Influence Of Circuit And Element Parasitics
Bessel Filter Final Realization

- De-Normalized Realization

- Branch Elements
  - Circuit Resistances Are In Units Of Ohms
  - Circuit Inductances Are In Units Of Nanohenries
  - Circuit Capacitances Are In Units Of Femtofarads
Simulated Bessel Filter I/O Responses

![Graph showing I/O gain vs. frequency for a Bessel filter. The graph plots the I/O gain (v/v) on the y-axis against frequency (Hz) on the x-axis. The graph shows a smooth transition from input to output with a decrease in gain as frequency increases.](image-url)
Simulated Bessel Filter Delay Response

![Graph of Simulated Bessel Filter Delay Response](image-url)
Simulated Bessel Filter Pulse Response

![Simulated Bessel Filter Pulse Response Graph](image)

- **TIME (s)**
- **TRANSIENT RESPONSES (V)**

- **V(OUT)**
- **V(IN)**

HSPICE
Comments On Bessel Simulations

- **Small Signal Response**
  - Flat Response -- No Peaking Over Signal Frequency
  - “DC” Gain Is 0.3317---Agrees
  - “DC” Gain At Input Port Is 0.8571, Independent Of Frequency, Which Corroborates With Constant 300 Ohm Input Impedance
  - I/O Envelope Delay Is Flat And Constant At 25 pSEC, To Within About 10%, Through The 3-dB Bandwidth Of Circuit

- **Transient Response**
  - Exceptionally Well-Behaved Response To Periodic Pulse Input
  - Miniscule Overshoot Evidenced At Rising Edges Of Pulses

- **Conclusion**
  - Design Strategy Works
  - Does Not Yield Optimal Circuit In Sense Of Topological Simplicity
  - Does Not Necessarily Yield Lossless Structure